MATH 432/532, MIDTERM PRESENTATIONS

Our midterm assessment will be through presentations you make on topics which build and elaborate on the fundamental group. Those additional topics are

(1) The Van Kampen Theorem.
(2) The Jordan separation and curve theorems.
(3) Embeddings of graphs.
(4) Classification of covering spaces.
(5) Classification of surfaces.

You will present them, as groups, starting on Wednesday February 2nd (Groundhog Day). Each group gets 25 minutes to present - 20 minutes of presentation and an extra 5 for questions. You will then write further elaborations on these topics individually, turning your write-ups in on February 11th.

In your presentations your first goals are to clearly define the relevant terms and state the main theorems, giving basic examples to illustrate them. You should then outline the proofs, highlighting the most important steps, especially as they pertain to the application or development of the fundamental group. In your write-ups, you should clearly develop the basic ideas which are in the book, but then expand to give additional material - examples, history, further theorems found in other texts. These write-ups should be roughly 3-6 pages in length.

To get you off to a good start, here are some ideas on what could go in the oral and written presentations.

- The Van Kampen Theorem - start with Theorem 59.1, as many groups will need this result. The rest of your oral presentation should just be devoted to understanding the statement of the theorem (classical version, Theorem 70.2), in particular just understanding the algebra of free products and amalgamated free products. One fun piece of algebra as an exercise - understand (for example, classify all subgroups of) $\mathbf{Z}/2 * \mathbf{Z}/2$. In the written piece, some history could be nice.

- The Jordan Theorems - In the oral presentation, go very carefully through Theorem 61.3 and do what you can with Theorem 63.4. For your written presentation, you might want to spice things up by discussing the Alexander horned sphere as an example of how things get complicated in higher dimensions (perhaps even touching on this at the end of the oral presentation as well).

- Embeddings of graphs - You should be able to do most of section 64 in your oral presentation (cut things out as needed). For your written presentation, you should also address the following questions: does the complete graph on five vertices embed on a sphere? on a torus? Can you give examples of graphs which do not embed on a sphere, giving as much of a proof as you can? Do the same for a torus. After trying your hand at things for a while, feel free to try to look up general results on embedding graphs.

- Classification of covering spaces - this is a rather vast topic. To narrow it down, first focus on the fact that if $p : X \to Y$ is a covering map then $p_\#$ is injective. Then simply state the big theorem that, if Y satisfies rather mild hypotheses (which you might not even be able to state) then there is a correspondence between isomorphism classes of connected covers of Y with a chosen basepoint and subgroups of $\pi_1(Y, y_0)$. Focus then on the example of $\mathbf{RP}^2 \vee \mathbf{RP}^2$ - you can work the Van Kampen on understanding $\mathbf{Z}/2 * \mathbf{Z}/2$. Once you understand the algebra, I will help you on making the covering spaces.

- Classification of surfaces - your main goal in the talk is to make the list of surfaces familiar, which requires developing the connected sum of surfaces and understanding better the projective plane and Klein bottle. One important piece in the classification is Exercise 4(a) in section 74; given this you can just say that all surfaces are produced by starting with a sphere and taking the connect sum with some number of tori and projective planes. Finally, highlight how with the Van Kampen Theorem you can compute the fundamental group of any surface and note (you won't have time to prove) that these fundamental groups distinguish all surfaces. For the written work, I'd be very interested in some history - this is one of those theorems which must have been proven non-rigorously well before topology was even a full-fledged subject.