(1) Define relative homology and the long exact sequence of a pair.

(2) Compute the relative homology groups $H_*(\mathbb{R}^2, \mathbb{Q}^2)$, giving an explicit basis in dimension one.

(3) Compute all the groups and homomorphisms in the long exact sequence for the pair (K, A) where K is the Klein bottle and A is the circle which is the image of the edges which are identified through an orientation reversing in the standard presentation of K as the quotient of a square.

(4) (a) Give the general definition (recursive or otherwise) of the barycentric subdivision of a single simplex $\Delta^n = [v_0, \ldots, v_n]$ as an element b_n^1 of $C_n(\Delta^n)$. Say what this is explicitly when $n = 2$.

(b) Let $\beta_n : C_n(X) \to C_n(X)$ be defined by sending a generator σ to $\sigma_\#(b_n^1)$. Explicitly construct the first two stages in a chain homotopy between β and the identity map.

(5) Use excision to show that $H_*(X, A) \cong \widetilde{H}_*(X/A)$ if (X, A) is a good pair.

(6) Finish off our computation of the homology of $\mathbb{RP}^n$ from last term, now that we know to compute using the long-exact sequence of the pair $(\mathbb{RP}^n, \mathbb{RP}^{n-1})$.

(7) Use Mayer-Vietoris to compute the homology of the space one obtains by taking n tori with an open disk removed and identifying all of the boundary circles (so one has a "book" of tori).

(8) Find the homology groups of the (singular) surface you get by rotating the circle $(x - \frac{1}{2})^2 + y^2 = 1$ about the y-axis.

(9) Define the mod 2 degree of the map using differential topology, outlining the proof that it is well-defined.

(10) Hatcher problems (page, #): (155, 4), (155, 8), (156, 14).