

TATE COHOMOLOGY OF THEORIES WITH ONE DIMENSIONAL COEFFICIENT RING.

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ABSTRACT. For a finite group G we calculate the E -Tate cohomology $t(E)_G^*$ and the E -homology $E_*(BG_+)$ as functors of the augmented commutative ring $E^*(BG_+)$ when $E^*(\cdot)$ is a complex oriented, v_n periodic cohomology theory with one dimensional graded coefficient ring E^* .

1. INTRODUCTION.

Let G be a finite group and p be a prime. Our aim is to calculate the coefficient ring of the G -equivariant Tate theory for a complex oriented v_n -periodic theory E . In [7] we showed that if E is mod p Morava K -theory (whose coefficient ring $K(n)^* = \mathbb{F}_p[v_n, v_n^{-1}]$ is a graded field) then the associated Tate theory is trivial and the representing spectrum is equivariantly contractible

$$t(K(n)) \simeq *.$$

The basis of this is Ravenel's result [11] that $K(n)^*(BG_+)$ is a finitely generated module over the graded field $K(n)^*$, and the simple form of the answer relies on the fact that $K(n)^*$ has Krull dimension 0 as a graded ring. In the present paper we consider the case when the coefficient ring E^* is of Krull dimension 1, thinking particularly of the case with coefficients

$$\mathbb{Z}_{(p)}[v_1, v_2, \dots, v_n, v_n^{-1}]/I_{n,i}$$

or a suitable completion, where $I_{n,i} = (p, v_1, \dots, \hat{v}_i, \dots, v_{n-1})$ is generated by all but one of the elements $v_0 = p, v_1, \dots, v_{n-1}$. For instance if $n = 1$ and $i = 0$ this gives the principal Adams summand of p -local K -theory. For all these theories $t(E)_G^*$ is nonzero and we give an exact calculation in (6.1).

We begin by explaining the importance of Krull dimension, and the place of the Tate theory in calculating $E_*(BG_+)$ from $E^*(BG_+)$. This also directs attention to the important features we shall need of E to be able to give definite answers.

2. PHILOSOPHY.

Let E be a complex oriented v_n -periodic theory and consider the ring $R^* = E^*(BG_+)$. Using the maps induced by $* \rightarrow BG \rightarrow *$ we see that R^* is an augmented E^* -algebra, and we shall have to consider the augmentation ideal $J^* = \ker(R^* \rightarrow E^*)$. Because E^* is periodic we may work with the degree 0 part of all rings and apply standard commutative algebra. To begin with, note that J^0 generates an ideal in R^* with the same radical as

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J^* . Indeed if $x \in J^*$ then $x^{2(p^n-1)}$ lies in a degree containing a power of the unit v_n , and is thus a unit multiple of an element of J^0 . From now on we use the abbreviations $R = R^0 = E^0(BG_+)$ and $J = J^0$.

The following method only works easily if J is finitely generated (at least up to taking radicals). In the present context we are content to assume rather more.

Finiteness Hypothesis 2.1. *The ring E^0 is Noetherian and the map $E^0 \longrightarrow R$ makes R into a finite E^0 -module.*

Under this hypothesis R is also Noetherian, and hence J is finitely generated. Furthermore, E^0 and R have the same Krull dimension, which we shall denote by d . We may therefore apply the standard constructions of commutative algebra to R . In particular, for any R -module M we may form the local cohomology groups $H_J^*(M)$ in the sense of Grothendieck [9]; see [3] for a summary of definitions in a convenient form. The essential property from our point of view is Grothendieck's vanishing theorem, which states that $H_J^i(M) = 0$ if $i > d$. The importance of this in equivariant topology comes from the local cohomology theorem which states there is a finite spectral sequence for calculating the homology of classifying spaces:

$$(2-2) \quad E_2^{s,t} = H_J^{s,t}(R^*) \implies E_{-(s+t)}(BG_+)$$

with differentials $d_r : E_r^{s,t} \longrightarrow E_r^{s+r,t-r+1}$. Some constructions give an E_2 term with J replaced by a subideal $J' \subseteq J$, but these ideals all have the same radical and hence [2, p. 297] the same local cohomology groups. For p -groups (2-2) was proved in [2], but we shall give an outline below. For general finite groups it may be proved using the machinery of highly structured ring spectra [1], although we do not need this case for our main result.

This gives us an interest in calculating the local cohomology groups of R^* . These are bigraded since R^* is graded, but because J is in degree 0 the local cohomology can be calculated one degree at a time in the sense that $H_J^{s,t}(R^*) = H_J^s(R^t)$. We simplify our notation accordingly.

It is common practice to calculate local cohomology groups using the Čech cohomology groups $\check{H}_J^*(R^*)$ together with the exact sequence

$$(2-3) \quad 0 \longrightarrow H_J^0(R^*) \longrightarrow R^* \longrightarrow \check{H}_J^0(R^*) \longrightarrow H_J^1(R^*) \longrightarrow 0$$

and the isomorphisms

$$(2-4) \quad H_J^i(R^*) \cong \check{H}_J^{i-1}(R^*) \text{ for } i \geq 2.$$

The advantages are twofold. Firstly, provided $d \geq 1$, the Čech groups vanish in degree d as well as above it (i.e. $\check{H}_J^i(R^*) = 0$ for $i \geq d$), and secondly if

$$\Gamma_J R^* = \{x \in R^* \mid J^N x = 0 \text{ for sufficiently large } N\}$$

denotes the J -power torsion ideal it is easy to see that $\check{H}_J^*(R^*) = \check{H}_J^*(R^*/\Gamma_J R^*)$, so that one may simplify calculations by factoring out the ideal $\Gamma_J R^*$ from the start.

The topological counterpart of Čech cohomology is the Tate theory for E in the sense of [5], indeed there is a spectral sequence

$$(2-5) \quad E_2^{s,t} = \check{H}_J^{s,t}(R^*) \implies t(E)_G^{s+t}$$

which collapses more often than (2-2) because of the vanishing theorem $\check{H}_J^d(R^*) = 0$ when $d \geq 1$. Again we shall construct the spectral sequence in case G is a p -group in 4.2 below, and this will suffice for our purposes, but it can be constructed for arbitrary finite groups by the methods of [1]. The topological counterpart of (2-3) is the norm sequence

$$(2-6) \quad \cdots \longrightarrow E_*(BG_+) \xrightarrow{\text{Norm}} E^*(BG_+) \longrightarrow t(E)_G^* \longrightarrow \cdots .$$

Thus our strategy in calculating $E_*(BG_+)$ is to use the local Tate spectral sequence (2-5) to calculate $t(E)_G^*$ and then the norm sequence (2-6).

We illustrate this in some cases with low Krull dimension below.

- $\dim E^0 = 0$: This is the case dealt with in [7] by using a topological argument. In the spirit of the present article we give a more algebraic proof using the local Tate cohomology of [3] in Section 8 below. In any case $t(E)_G^* = 0$, and the norm gives an isomorphism $E_*(BG_+) \cong E^*(BG_+) = R^*$.
- $\dim E^0 = 1$: This is the principal case we deal with in the present note. Here the Čech cohomology is concentrated in degree zero and we have

$$t(E)_G^* = \check{H}_J^0(R^*).$$

To explain the resulting calculation of $E_*(BG_+)$ we need to be clear about grading. We regard all objects as graded in the cohomological fashion. Homology theories are given cohomological grading by taking the n th grading $(E_*(X))^n = E_{-n}(X)$ in the standard way. All suspensions refer to this cohomological grading so that $(\Sigma R^*)^n = R^{n-1}$. With these conventions the norm sequence gives an exact sequence

$$0 \longrightarrow \text{cok} \left\{ \Sigma R^* \longrightarrow \check{H}_J^0(\Sigma R^*) \right\} \longrightarrow E_*(BG_+) \longrightarrow \Gamma_J R^* \longrightarrow 0.$$

In particular if R^* is concentrated in even degrees

$$E_{\text{even}}(BG_+) = \Gamma_J R^* \text{ and } E_{\text{odd}}(BG_+) = \text{cok} \left\{ \Sigma R^* \longrightarrow \check{H}_J^0(\Sigma R^*) \right\}.$$

- $\dim E^0 = 2$: If R^* is concentrated in even degrees then

$$t(E)_G^{\text{even}} = \check{H}_J^0(R^*) \text{ and } t(E)_G^{\text{odd}} = \check{H}_J^1(\Sigma R^*),$$

and the norm sequence gives

$$0 \longrightarrow \check{H}_J^1(R^*) \longrightarrow E_{\text{even}}(BG_+) \longrightarrow \Gamma_J R^* \longrightarrow 0$$

and

$$0 \longrightarrow \Sigma R^* / \Gamma_J R^* \longrightarrow \check{H}_J^0(\Sigma R^*) \longrightarrow E_{\text{odd}}(BG_+) \longrightarrow 0.$$

- $\dim E^0 = 3$: If R^* is concentrated in even degrees the local Tate spectral sequence (2-5) collapses to give

$$t(E)_G^{\text{odd}} = \check{H}_J^1(\Sigma R^*)$$

and an exact sequence

$$0 \longrightarrow \check{H}_J^2(\Sigma^2 R^*) \longrightarrow t(E)_G^{\text{even}} \longrightarrow \check{H}_J^0(R^*) \longrightarrow 0.$$

In this case there is a possible differential $d_3 : H_J^0(R^*) \longrightarrow H_J^3(\Sigma^2 R^*)$ in the local cohomology spectral sequence (2-2), and the image of the norm map $E_*(BG_+) \longrightarrow E^*(BG_+) = R^*$ is $\ker(d_3)$, which may therefore be a proper submodule of $\Gamma_J R^*$.

- $\dim E^0 = 4$: If R^* is concentrated in even degrees there is only one possible differential in the local Tate spectral sequence (2-5) so that we obtain an exact sequence

$$0 \longrightarrow \check{H}_J^2(\Sigma^2 R^*) \longrightarrow t(E)_G^{\text{even}} \longrightarrow \check{H}_J^0(R^*) \xrightarrow{d_3} \check{H}_J^3(\Sigma^2 R^*) \longrightarrow t(E)_G^{\text{odd}} \longrightarrow \check{H}_J^1(R^*) \longrightarrow 0.$$

It would be interesting to understand this first differential.

3. FINITENESS

In this section we establish a basic finiteness result for suitably complete theories. The finiteness does not hold for arbitrary v_n -periodic theories as we shall see in Section 7, but the finiteness for complete theories is sufficient to let us proceed more generally.

Let E_n be the completion of the Johnson-Wilson theory $E(n)$ at (p, v_1, \dots, v_{n-1}) made 2-periodic. If $n = 1$ we obtain p -adic K-theory, and in general E_n has coefficients

$$E_n^* = \mathbb{Z}_p^\wedge[[u_1, u_2, \dots, u_{n-1}]] [u, u^{-1}],$$

where u_i is of degree 0 and u is of degree -2 . These generators are related to the v_i by $v_i = u^{1-p^i} u_i$ for $1 \leq i \leq n-1$ and $v_n = u^{1-p^n}$. The advantages of using this theory will be apparent when we give a very simple example with the uncompleted theory in Section 7 below.

Theorem 3.1. *For any finite group G , $E_n^*(BG_+)$ is a finite module over E_n^* .*

Notice that $E(1)^*(BG_+)$ is a sum of $E(1)^* = \mathbf{Z}_{(p)}[v_1, v_1^{-1}]$ and a number of copies of $E(1)_p^* = \mathbf{Z}_p^\wedge[v_1, v_1^{-1}]$ by the Atiyah-Segal completion theorem, so it is not finite over $E(1)^*$ unless G is trivial. This shows some form of completion is necessary for such a finiteness theorem to hold.

We shall prove 3.1 by induction on the number of spectral lines in the coefficients (i.e. the number of polynomial generators v_j or u_j), starting from Ravenel's finiteness theorem [11].

Let E be a ring spectrum and $v : E \rightarrow E$ a map of spectra induced by an element $v \in E^*$. In our applications this element will be a power of v_i . Let E/v^k denote the cofibre of $v^k : E \rightarrow E$, and let

$$\hat{E} = \varprojlim_k E/v^k.$$

Finally let $K = E/v$.

Proposition 3.2. *Suppose that K^* is Noetherian and that $\pi_r(E/v^k)$ is a compact topological space for all r, k , such that the maps $\pi_k(E/v^k) \rightarrow \pi_k(E/v^{k-1})$ are continuous. If X is a locally finite, bounded below spectrum and $K^*(X)$ is finitely generated as a module over K^* , then $\hat{E}^*(X)$ is finitely generated as a module over \hat{E}^* .*

Proof. The proof is based on the Bockstein spectral sequence. We form the tower

$$\dots \rightarrow E/v^3 \rightarrow E/v^2 \rightarrow E/v = K \rightarrow *.$$

Of course the cofibres at each stage are just copies of K . If we map X into the tower, we get in the usual way a Bockstein spectral sequence

$$E_1 = K^*(X) \otimes_{K^*} K^*[v] \Rightarrow \widehat{E}^*(X).$$

Our compactness assumptions guarantee convergence. First note that $\widehat{E}/v^k \simeq E/v^k$ for all k . It follows that $[T, \widehat{E}/v^k]$ is compact for any finite spectrum T . Now we calculate

$$\begin{aligned} [X, \widehat{E}] &= \varprojlim_n [X^{(n)}, \widehat{E}] = \varprojlim_n [X^{(n)}, \varprojlim_k E/v^k] \\ &= \varprojlim_n \varprojlim_k [X^{(n)}, E/v^k] = \varprojlim_k [X, E/v^k]. \end{aligned}$$

The first equality follows since X is locally finite and the homotopy of \widehat{E} is compact. The second equality is a definition. The third follows from the assumption about the compactness of $\pi_k(E/v^k)$ and the finiteness of $X^{(n)}$ giving compactness of $[X^{(n)}, E/v^k]$. The fourth commutes some limits and uses the compactness of the homotopy of E/v^k again.

Returning to the argument, let $\overline{E}_1 \subseteq E_1$ be the subgroup of permanent cycles. Here we make use of the only nontrivial property of the Bockstein spectral sequence. The differentials commute with multiplication by v in the sense that $d_r(vx) = vd_r(x)$. To see this we note that we may form a second copy of the original tower related by multiplication by v . It follows that \overline{E}_1 is actually a sub- $K^*[v]$ -module of E_1 .

By hypothesis E_1 is finitely generated over $K^*[v]$; K^* is Noetherian so is $K^*[v]$. Therefore \overline{E}_1 is finitely generated over $K^*[v]$. It follows that the quotient, E_∞ is also finitely generated over $K^*[v]$. Thus we have an associated graded module finitely generated over the associated graded ring, and both are complete for their respective filtrations. The usual argument about lifting generators tells us that the original module, $\widehat{E}^*(X)$ is finitely generated over the original ring \widehat{E}^* . \square

Now we can apply the proposition to prove the theorem. We use the theories $E_{n,i}$ with coefficients

$$E_{n,i}^* = E_n^*/(p, v_1, \dots, v_{n-1-i}),$$

interpolating between $E_{n,0} = K(n)$ and $E_{n,n} = E_n$. Now we argue by induction on i , taking $E = E_{n,i+1}$, $K = E_{n,i}$ and $v = v_{n-i-1}$. Ravenel's finiteness theorem supplies the base for the induction, and since E_n is complete with respect to (p, v_1, \dots, v_{n-1}) we have $E = \widehat{E}$ at each stage. \square

4. REDUCTION TO EULER CLASSES.

Our method of calculation relies on the local Tate spectral sequence for calculating $t(E)_G^*$ from $E^*(BG_+)$ with various Euler classes inverted. We take advantage of the fact that the Tate spectrum of a split G -spectrum depends only on the underlying nonequivariant spectrum. This means that whatever the best model for equivariant E -cohomology, it has the same Tate spectrum as the Borel theory $X \mapsto E^*(EG_+ \wedge_G X)$ represented by

$F(EG_+, E)$. We simply write $t(E)$ for the G -spectrum representing this Tate theory, and the homological description is

$$t(E) = F(EG_+, E) \wedge \tilde{E}G.$$

Thus any good model for $\tilde{E}G$ gives rise to a means of calculating $t(E)_G^*$ from $R^* = E^*(BG_+) = \pi_*^G(F(EG_+, E))$.

For instance, if G acts freely on the unit sphere $S(V)$ of a complex representation V we find $S(\infty V) = \bigcup_k S(kV)$ is a model for EG . Since the one point compactification S^V of V is the mapping cone of $S(V)_+ \rightarrow S^0$, we find $S^{\infty V} = \bigcup_k S^{kV}$ is a model for $\tilde{E}G$. In general if G acts freely on $S(V_1) \times S(V_2) \times \cdots \times S(V_n)$ for some n , the suspension spectrum of $\tilde{E}G$ admits a filtration so that for $s = 0, 1, \dots, n-1$ the s th subquotient is the s -fold desuspension of the wedge of all $(s+1)$ -fold smash products $S^{\infty V_{i(0)}} \wedge S^{\infty V_{i(1)}} \wedge \cdots \wedge S^{\infty V_{i(s)}}$ where $i(0) < i(1) < \cdots < i(s)$. To see this, one notes that $\Sigma S(\infty V)_+ = S^{\infty V} \cup CS^0$, so that $EG_+ = S(\infty V_1)_+ \wedge S(\infty V_2)_+ \wedge \cdots \wedge S(\infty V_n)_+$ admits the product filtration with top quotient S^0 and fibre $\Sigma^{-1}\tilde{E}G$. Explicitly, in the case where G acts freely on $S(V) \times S(W)$ then $S(\infty V) \times S(\infty W)$ is a model for EG and there is a stable cofibre sequence $\tilde{E}G \rightarrow S^{\infty V} \vee S^{\infty W} \rightarrow S^{\infty V \oplus W}$.

Therefore once we can calculate $\pi_*^G(F(EG_+, E) \wedge S^{\infty V})$, we have a means to calculate $t(E)_G^*$ whenever G acts freely on a product of spheres.

Lemma 4.1. *For any complex representation V with Euler class $\chi(V) \in E^{|V|}(BG_+)$ we have*

$$\pi_*^G(F(EG_+, E) \wedge S^{\infty V}) = R^*[1/\chi(V)].$$

Proof. For any G -spectrum X we have $\pi_*^G(X \wedge S^{\infty V}) = \varinjlim_k \pi_*^G(X \wedge S^{kV})$. If X is a ring spectrum with Thom isomorphisms for V then the image of the unit from $\pi_0^G X$ in the diagram

$$\begin{array}{ccc} \pi_n^G(X \wedge S^0) & \xrightarrow{e(V)_*} & \pi_n^G(X \wedge S^V) \\ \cong \downarrow & & \downarrow \cong \\ \pi_n^G(X) & \xrightarrow{\chi(V)} & \pi_{n-|V|}^G(X) \end{array}$$

defines the Euler class $\chi(V) \in \pi_{-|V|}^G(X)$. One checks that $\chi(kV) = \chi(V)^k$, and hence

$$\pi_*^G(X \wedge S^{\infty V}) = \varinjlim_k \{\pi_*^G(X), \chi(V)\} = \pi_*^G(X)[1/\chi(V)]$$

as required. \square

It will again be useful to use the periodicity of E^* to shift the Euler classes into degree 0, as is familiar for equivariant K -theory. Replacing V by a multiple if necessary, we may suppose that the real dimension of V is a multiple of the degree of v_n . Thus $\chi(V)$ is a v_n -multiple of an element $\lambda(V)$ in degree 0, so that inverting $\chi(V)$ has the same effect as inverting $\lambda(V)$.

Now for any elements x_1, x_2, \dots, x_n in R we may form the Čech complex

$$\bigoplus_{i(0)} R[1/x_{i(0)}] \longrightarrow \bigoplus_{i(0) < i(1)} R[1/(x_{i(0)}x_{i(1)})] \longrightarrow \cdots \longrightarrow \bigoplus_{i(0) < i(1) < \cdots < i(s)} R[1/(x_{i(0)}x_{i(1)} \cdots x_{i(s)})].$$

It is not hard to see that the cohomology only depends on the radical of the ideal $I = (x_1, \dots, x_n)$ [2]; this cohomology is the Čech cohomology $\check{H}_I^*(R)$. By construction we therefore have a local Tate spectral sequence in case G acts freely on a product of spheres.

Corollary 4.2. *If G acts freely on a product of spheres and $J' = (\lambda(V_1), \dots, \lambda(V_n))$ is the ideal generated by their degree 0 Euler classes there is a spectral sequence*

$$E_2^{s,*} = \check{H}_{J'}^s(R^*) \implies t(E)_G^*.$$

If J' has the same radical as J then the E_2 term is $\check{H}_J^s(R^)$.* \square

In fact, if R^* is Noetherian, the augmentation ideal has the same radical as the Euler class ideal, so that this is consistent with (2-5). More precisely, we say that the cohomology theory $E_G^*(\cdot)$ is Noetherian if the coefficient ring is Noetherian, and $E_G^*(X)$ is a finitely generated module whenever X is a finite G -CW complex. The following result is a special case of Theorem 1.4 of [4].

Theorem 4.3. *If $F^*(\cdot)$ is a Noetherian cohomology theory with F^* an integral domain, and if G acts freely on $S(V_1) \times \dots \times S(V_n)$ then the radical of the ideal $J' = (\lambda(V_1), \dots, \lambda(V_n))$ is the augmentation ideal $J(G)$.* \square

Henceforth we shall use the augmentation ideal rather than J' without further comment. The reader content to restrict attention to p -groups can work entirely with the more concrete Euler class ideal J' , and need never invoke this comparison theorem. The calculational consequences are identical. However, even for p -groups the augmentation ideal seems more natural, and for general finite groups explicit substitutes for it are clumsier than J' . The theorem is also essential when comparing the ideals for different groups under restriction.

5. INVERTING EULER CLASSES

We have now established that the calculation depends on an understanding of R localized away from the Euler classes. The key statement we need is that inverting an Euler class $\lambda(V) \in R$ is essentially the same as inverting an element $\bar{\lambda}(V)$ of the ground ring E^0 . The geometric flavour of this is that the map of varieties associated to $E^0 \rightarrow R$ is injective on the hypersurface defined by any Euler class. This geometric point of view is developed further in [8]. As a special bonus the element $\bar{\lambda}^*(V)$ is independent of V in the one dimensional case.

In this section E^* has a single spectral line v_i for some $0 \leq i \leq n-1$; for example the coefficients may be $\mathbb{Z}_{(p)}[v_1, v_2, \dots, v_{n-1}, v_n, v_n^{-1}]/(p, v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_{n-1})$ or some completion. As in the introduction, we abbreviate the ideal $(p, v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_{n-1})$ by $I_{n,i}$.

Let $a(V) = \ker(R^* \rightarrow R^*[1/\chi(V)])$ and $Q(V) = R^*/a(V)$. It is immediate that $R^*[1/\chi(V)] = Q(V)[1/\chi(V)]$.

Theorem 5.1. *If $E = E(n)/I_{n,i}$ or $E_n/I_{n,i}$ and $V \neq 0$ then v_i is invertible on $R^*[1/\chi(V)]$ for any complex representation V ; if $E = E_n$ then*

$$R^*[1/\chi(V)] = Q(V)[1/v_i].$$

The expectation is that v_i should be invertible for any theory with a single spectral line, and that if it is also complete then one has a precise identification of $R^*[\chi(V)]$. It is clear that the method of proof below will work in many other cases, but the correct level of generality is not clear.

Proof. The proof is in two steps; firstly we show that v_i is invertible on $R^*[1/\chi(V)]$, and secondly that in the complete case $Q(V)[1/v_i]$ is finite enough that any monomorphism is an isomorphism.

Lemma 5.2. *The element v_i is invertible on $R^*[1/\chi(V)]$.*

Proof. Because the Euler for $K(n)^*$ is the image of the Euler class from E^* , the cofibre sequence $E \xrightarrow{v_i} E \rightarrow K(n)$ gives a long exact sequence

$$\cdots \rightarrow R^*[1/\chi(V)] \xrightarrow{v_i} R^*[1/\chi(V)] \rightarrow K(n)^*(BG_+)[1/\chi(V)] \rightarrow \cdots$$

But $K(n)^*(BG_+)[1/\chi(V)] = 0$ by Lemma 2.2 of [7]. \square

Now $\chi(V) : Q(V) \rightarrow Q(V)$ is injective, and therefore so is $\chi(V) : Q(V)[1/v_i] \rightarrow Q(V)[1/v_i]$. Furthermore, $Q(V)$ is finite over E^0 so $Q(V)[1/v_i]$ is finite over $E^0[1/v_i]$.

The completeness assumption on E is necessary to obtain the conclusion of the following lemma.

Lemma 5.3. *For $E = E_n/I_{n,i}$, $E^*[1/v_i]$ is a graded field.*

Proof. In fact it is easy to verify that for any graded field k then for an indeterminate y the ring $k[[y]][1/y]$ is again a graded field. \square

Therefore the monomorphism $\chi(V) : Q(V)[1/v_i] \rightarrow Q(V)[1/v_i]$ is also an epimorphism. \square

6. THE PROOF.

In this section we prove our main theorem. Note in particular that despite the necessity of completeness hypotheses in the previous section, none is necessary here. This is because although R^* need not be v_i -complete, the relevant quotient $R^*/\Gamma_J R^*$ is always v_i -complete.

For a higher dimensional v_n -periodic cohomology theory one may ask whether $R^*/\Gamma_J R^*$ is still I_n -complete. This is false in general as the referee kindly pointed out; take our theory to be the Johnson-Wilson theory $E(2)$. Take $G = \mathbf{Z}/(2)$. Then $R^* = E(2)^*[[x]]/([2](x))$ and $\Gamma_J R^*$ is the $E(2)^*$ -module generated by $[2](x)/x$. Now modulo 2,

$$[2](x) = v_1 x^2 +_F v_2 x^4 \text{ so } [2](x)/x = xp(x)$$

where $p(x)$ is still some power series in x . So $[R^*/\Gamma_J R^*]/(2) = \mathbf{F}_2[v_1][v_2, v_2^{-1}][[x]]/xp(x)$ and modulo (x) , $[R^*/\Gamma_J R^*]/(2, x) = \mathbf{F}_2[v_1][v_2, v_2^{-1}]$ which is not complete with respect to v_1 , so is not I_2 -adically complete.

Theorem 6.1. *If $E = E(n)/I_{n,i}$ or $E_n/I_{n,i}$ then*

$$t(E)_G^* = (R^*/\Gamma_J R^*)[1/v_i]$$

where $\Gamma_J R^*$ is the ideal of J -power torsion elements of R^* .

Corollary 6.2. *If E is as in the theorem and R^* is concentrated in even degrees then*

$$E_{\text{even}}(BG_+) = \Gamma_J R^* \text{ and } E_{\text{odd}}(BG_+) = \Sigma\{(R^*/\Gamma_J R^*)/v_i^\infty\}.$$

Again one expects this behaviour for any complex oriented v_n -periodic theory with a single spectral line, and the given proof will work in many cases of this type.

Proof. We shall begin by proving the theorem when E is complete, and then explain why the general result follows.

If G is a p -group it acts freely on a product of linear spheres and hence by Corollary 4.2 it is enough to show $\check{H}_J^0(R^*) = (R^*/\Gamma_J R^*)[1/v_i]$. The general case will follow from a transfer argument.

The exact sequence (2-3) gives a short exact sequence

$$0 \longrightarrow R^*/\Gamma_J R^* \longrightarrow \check{H}_J^0(R^*) \longrightarrow H_J^1(R^*) \longrightarrow 0.$$

From Theorem 5.1, v_i is invertible on $\check{H}_J^*(R^*)$, so that we obtain an injective map

$$(R^*/\Gamma_J R^*)[1/v_i] \longrightarrow \check{H}_J^0(R^*).$$

The cokernel is the group $H_J^1(R^*)[1/v_i] = H_J^1(R^*[1/v_i])$, which is zero since $R^*[1/v_i]$ (or rather its degree 0 subring) is 0-dimensional; this is where we use completeness. This proves the theorem when G is a p -group.

Now for a general group G with Sylow p -subgroup P we first need the fact that $t(E)_G^*$ is a retract of $t(E)_P^*$.

Lemma 6.3. *If G is a group with Sylow p -subgroup P and E is a p -local G -spectrum then $t(E)_G^*$ is a retract of $t(E)_P^*$.*

Proof. We work with G -spectra and use the map $\pi : G/P_+ \longrightarrow S^0$ and its dual $D\pi : S^0 \longrightarrow DG/P_+ \simeq G/P_+$ to construct the diagram in which the horizontals are cofibre sequences

$$\begin{array}{ccccc} E \wedge EG_+ & \longrightarrow & F(EG_+, E) & \longrightarrow & t(E) \\ \downarrow & & \downarrow & & \downarrow \\ G/P_+ \wedge E \wedge EG_+ & \longrightarrow & G/P_+ \wedge F(EG_+, E) & \longrightarrow & G/P_+ \wedge t(E) \\ \downarrow & & \downarrow & & \downarrow \\ E \wedge EG_+ & \longrightarrow & F(EG_+, E) & \longrightarrow & t(E). \end{array}$$

We argue that the first and second vertical composites are equivalences, and deduce that the third is by the 5-lemma. Hence the G -spectrum $t(E)$ is a retract of $G/P_+ \wedge t(E)$; the result follows by taking homotopy groups.

From the identification of $[S^0, S^0]^G$ with the Burnside ring $A(G)$ the vertical composites are multiplication by $[G/P] \in A(G)$. Now $[G/P]$ acts as multiplication by the index $|G : P|$ on both $G/1_+ \wedge E$ and $F(G/1_+, E)$; since E is p -local this is an equivalence. Now any free spectrum T can be constructed out of cells $G/1_+ \wedge S^n$ and hence using wedges, cofibre sequences and limits we see that $[G/P]$ is an equivalence of $T \wedge E$ and $F(T, E)$. Taking $T = EG_+$ we obtain the required result. \square

Since the retractions are all maps of E_* -modules it follows that v_i acts invertibly on $t(E)_G^*$. From the lemma we obtain a diagram

$$\begin{array}{ccccc} E_*(BG_+) & \longrightarrow & R_G^* & \xrightarrow{\alpha} & t(E)_G^* \\ \downarrow & & \downarrow & & \downarrow \\ E_*(BP_+) & \longrightarrow & R_P^* & \longrightarrow & t(E)_P^* \end{array}$$

which is part of a long exact ladder, and in which the verticals are injective and the four rightmost maps are ring homomorphisms. From the case of the p -group P we see that inverting v_i in the diagram gives a short exact sequence in the second row, and hence also in the first row. It therefore suffices to identify the kernel K of α with $\Gamma_{J(G)}R_G^*$. Now by [4, 1.2], since R_G^* and R_P^* are Noetherian, $\sqrt{\text{res}_P^G(J(G))} = J(P)$, and hence the two ideals have the same torsion functors: $\Gamma_{J(G)}M = \Gamma_{J(P)}M$ for any R_P^* -module M . Now $K = R_G^* \cap \Gamma_{J(P)}R_P^* = R_G^* \cap \Gamma_{J(G)}R_P^* = \Gamma_{J(G)}(R_G^*)$ as required.

This finishes the proof in case E is complete.

For the non-complete case, let $E^\wedge = \varprojlim_k E/v_i^k$, denote its completion, and similarly let $R^* = E^*(BG_+)$ and $(R^\wedge)^* = (E^\wedge)^*(BG_+)$. Now we have a ring homomorphism $R^* \longrightarrow (R^\wedge)^*$, and all $(R^\wedge)^*$ -modules can be regarded as R^* -modules by pullback. Next we claim that if J is the augmentation ideal of R , and J^\wedge of R^\wedge , then the image of J has the same radical as J^\wedge ; this allows us to use the results for the complete case. If G is a p -group, this follows from [4, 1.4] since the ideal J contains all Euler classes, and in the general case it follows since $J(G) = J(P) \cap E^*(BG_+)$. Thus local and Čech cohomology at J^\wedge of $(R^\wedge)^*$ -modules can be calculated by regarding them as R^* -modules and using J .

The main input is that the natural map

$$(6-4) \quad t(E) \xrightarrow{\simeq} t(E^\wedge)$$

is an equivalence. Indeed, the cofibre is $t(E')$, where E' is the cofibre of $E \longrightarrow E^\wedge$. This cofibre is $\varprojlim (E, v_i)$, so by construction v_i is an isomorphism on E' . If we localize by inverting v_i , we get the cofibration

$$(6-5) \quad v_i^{-1}E \rightarrow v_i^{-1}E^\wedge \rightarrow E'.$$

By [7, 1.5], $t(v_i^{-1}E) \simeq t(v_i^{-1}E^\wedge) \simeq *$, and we conclude that $t(E') \simeq *$ as required.

Note that, since by the local cohomology theorem (2-2) $E_*(BG_+)$ is J -power torsion, the norm map $E_*(BG_+) \longrightarrow E^*(BG_+) = R^*$ factors through $\Gamma_J R^*$. Consider then the map $n : E_*(BG_+) \longrightarrow \Gamma_J R^*$. We argue below that it is surjective, so that we have an exact sequence $0 \longrightarrow R^*/\Gamma_J R^* \longrightarrow t(E)_G^* \longrightarrow \Sigma^{-1} \ker(n) \longrightarrow 0$. We also show that n becomes an isomorphism when we invert v_i . The result follows since v_i is invertible on $t(E)_G^*$ by [7, 1.5]. Alternatively, it can be deduced from (5.1) together with (4.2).

Consider the diagram

$$\begin{array}{ccccc} E \wedge EG_+ & \longrightarrow & E^\wedge \wedge EG_+ & \longrightarrow & E' \wedge EG_+ \\ \downarrow & & \downarrow & & \downarrow \\ F(EG_+, E) & \longrightarrow & F(EG_+, E^\wedge) & \longrightarrow & F(EG_+, E'), \end{array}$$

and apply homotopy groups to obtain a diagram

$$\begin{array}{ccccccc}
 \dots & \xrightarrow{\alpha} & E_*(BG_+) & \xrightarrow{\beta} & E^*(BG_+) & \xrightarrow{\gamma} & E'_*(BG_+) & \xrightarrow{\Sigma^{-1}\alpha} & \dots \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 \dots & \xrightarrow{\theta} & R^* & \xrightarrow{\phi} & (R^\wedge)^* & \xrightarrow{\psi} & (R')^* & \xrightarrow{\Sigma^{-1}\theta} & \dots
 \end{array}$$

Suppose $x \in \Gamma_J R^*$. Then x maps to 0 in $t(E)_*^G$ since it must map to 0 in $t(E^\wedge)_*^G$ and these are isomorphic by (6-4). This establishes that $n : E_*(BG_+) \rightarrow \Gamma_J R^*$ is surjective.

Finally, we show that n is injective after inverting v_i . First note that $v_i^{-1}(E_* BG_+) = (v_i^{-1}E)_* BG_+$, and that there is a natural map $v_i^{-1}(E^* BG_+) \rightarrow (v_i^{-1}E)^* BG_+$. Thus we have a sequence of maps

$$(v_i^{-1}E)_* BG_+ = v_i^{-1}(E_* BG_+) \rightarrow v_i^{-1}\Gamma_J R^* \rightarrow v_i^{-1}(E^* BG_+) \rightarrow (v_i^{-1}E)^* BG_+.$$

The composite is an isomorphism by [7, 4.1], and hence the map $v_i^{-1}(E_* BG_+) \rightarrow v_i^{-1}\Gamma_J R^*$ must be an injection. \square

7. EXAMPLES FOR ELEMENTARY ABELIAN GROUPS.

In this section we calculate $t(E)_G^*$ when G is elementary abelian in a more explicit form than given above. There are two classes of examples that we consider. Firstly we consider the complete 2-periodic theories $E = E_n/(p, u_1, \dots, \hat{u}_i, \dots, u_{n-1})$, where $E_n^* = \mathbb{Z}_p^\wedge[[u_1, \dots, u_{n-1}]]\langle u, u^{-1} \rangle$, as in Section 3. Secondly, by way of contrast we include a calculation for the theory $E = E(n)/(p, v_1, \dots, \hat{v}_i, \dots, v_{n-1})$. For convenience we refer to these as the 2-periodic and $2(p^n - 1)$ -periodic cases.

We begin by writing $E^*(B(C_{r_1} \times \dots \times C_{r_k})_+)$ explicitly. Let E be complex oriented, and assume that the r -series, $[r](x)$ for the formal group law F over E is not a 0-divisor in $E^*[[x]]$. By Lemma 4.2 of [6],

$$E^*([CP^\infty]_+^{\times k}) = E^*[[x_1, \dots, x_k]].$$

Now assume by induction (the base case is $i = 0$) that

(7-6)

$$E^*(BC_{r_1} \times \dots \times BC_{r_i} \times [CP^\infty]_+^{k-i}) = E^*[[x_1, \dots, x_k]]/([r_1](x_1), \dots, [r_i](x_i)).$$

Then since $[r_{i+1}](x_{i+1})$ is not a 0-divisor, by the Gysin sequence for

$$S^1 \rightarrow BC_{r_1} \times \dots \times BC_{r_i} \times [CP^\infty]_+^{k-i} \rightarrow BC_{r_1} \times \dots \times BC_{r_{i+1}} \times [CP^\infty]_+^{k-i-1}$$

we have (7-6) with $i + 1$ in place of i . Then by induction we have

$$E^*(B(C_{r_1} \times \dots \times C_{r_k})_+) = E^*[[x_1, \dots, x_k]]/([r_1](x_1), \dots, [r_k](x_k)).$$

Now let E be as in the first paragraph of this section. Since E is p -local, we'll assume the complex orientation factors through BP , as usual, and hence that $[p](x) = v_i x^{p^i} +_f v_n x^{p^n}$ [12, A2.2.4]. As shorthand we write $S = E^*((BC_p)_+) = E^*[[x]]/([p](x))$.

Lemma 7.1. *In S we have $x^{p^n} = u \cdot v_i x^{p^i} / v_n$ where u is a unit of degree zero in S .*

Proof. In $E^*[[x]]$ we find $[p](x) = v_i x^{p^i} + v_n x^{p^n} \pmod{v_i v_n x^{p^i+p^n}}$, and hence $v_n x^{p^n} = -v_i x^{p^i} (1 + x \cdot \alpha)$ for some α . \square

Thus any multiple of x^{p^n} may be expressed as a multiple of a lower power of x by a power series in v_i , and as modules over E^*

$$(7-2) \quad S = E^* \{1, x, \dots, x^{p^i-1}\} \oplus (E^*)_{v_i}^\wedge \{x^{p^i}, \dots, x^{p^n-1}\}.$$

Here the completion at v_i arises since power series in x have been converted to power series in v_i using the lemma.

It is worth noting some general features of this expression. Firstly S is concentrated in even degrees. Next observe that it is not usually finite over E^* unless E^* is v_i -complete. Finally notice that the degrees of the generators are in different congruence classes modulo $2(p^n - 1)$ except that 1 and x^{p^n-1} both lie in the congruence class of 0. The expression also shows that the first summand does not generally disappear if we replace BG_+ by BG , unless $i = 0$. This should provide a nonequivariant cure for any temptation the reader may have to delete the basepoint; working equivariantly provides another cure.

Now since C_p acts freely on $S(\eta)$ where η is a faithful one dimensional representation, and since x is the Euler class of any faithful one dimensional representation, we see that $t(E)_{C_p}^* = S[1/x]$ (where $M[1/x]$ will denote the S -module M localized with respect to the multiplicatively closed set generated by x). If we now let A denote the first summand in (7-2), and B the second then $x^{p^i} A \subseteq B$ and hence $(S/B)[1/x] = 0$. Thus there is an isomorphism $B[1/x] \xrightarrow{\cong} S[1/x]$, which completes the calculation for $G = C_p$. Note that the ideal $\Gamma_J R^*$ is generated by the elements $x^{p^n-1} - uv_i x^{p^i-1}/v_n, x^{p^n-2} - uv_i x^{p^i-2}/v_n, \dots, x^{p^n-p^i} - uv_i/v_n$. Thus

$$t(E)_{C_p}^* = B[1/x] = v_i^{-1} (E^*)_{v_i}^\wedge [1/v_i][x] / (x^{p^n-p^i} - uv_i/v_n) = (E^*)_{v_i}^\wedge [1/v_i] \{1, x, \dots, x^{p^n-p^i-1}\}$$

where the last isomorphism gives $t(E)_{C_p}^*$ additively as a free module over $(E^*)_{v_i}^\wedge [1/v_i]$.

If G is of rank r we may choose an isomorphism $G \cong H_1 \times H_2 \times \dots \times H_r$ where $H_i \cong C_p$. Consider $R^* = E^*(BG_+)$, and note from above that $R^* \cong S_1 \hat{\otimes} S_2 \hat{\otimes} \dots \hat{\otimes} S_r$ [10]. Now since H_i acts freely on $S(\eta_i)$ where η_i is a faithful one dimensional representation, G acts freely on $S(\eta_1) \times S(\eta_2) \times \dots \times S(\eta_r)$ and so $t(E)_G^*$ may be calculated from Čech complex for x_1, x_2, \dots, x_r . Since every term has at least one x_i inverted (which kills A_i) the result is the same as if E was replaced by its v_i -completion at the start. Since R is finite over E^0 in the complete case, and hence of dimension 1, the cohomology of the Čech complex is concentrated in degree zero and we find an exact sequence

$$0 \longrightarrow t(E)_G^* \longrightarrow \bigoplus_{0 \leq j \leq r} R^*[1/x_j] \longrightarrow \bigoplus_{1 \leq k < l \leq r} R^*[1/x_k x_l].$$

On the other hand, purely as a matter of combinatorics we know there is an exact sequence

$$0 \longrightarrow A_1 \otimes A_2 \otimes \dots \otimes A_r \longrightarrow R^* \longrightarrow \bigoplus_{0 \leq j \leq r} R_j^* \longrightarrow \bigoplus_{0 \leq k < l \leq r} R_{k,l}^*,$$

where R_j^* is the part of R^* with the j coordinate constrained to be B_j and similarly for $R_{k,l}^*$. Inverting v_i and mapping the lower sequence to the upper leads to the following calculation.

Theorem 7.2. *The coefficient ring of E -Tate theory is*

$$t(E)_G^* = \{R^*/(A_1 \otimes A_2 \otimes \cdots \otimes A_r)\}[1/v_i].$$

In particular it is concentrated in even degrees and it is free of finite rank $p^{nr} - p^{ir}$ over $(E^)_{v_i}^\wedge[1/v_i]$.*

If one desires explicit generators, one can take monomials in the $x_1^{t_1} \dots x_r^{t_r}$ such that $0 \leq t_i \leq p^n - 1$ and for some j , $t_j \geq p^i$.

Corollary 7.3. *The homology of BG_+ is as follows*

$$E_{\text{even}}(BG_+) = A_1 \otimes A_2 \otimes \cdots \otimes A_r$$

and

$$E_{\text{odd}}(BG_+) = \Sigma\{R/(A_1 \otimes A_2 \otimes \cdots \otimes A_r)\}/v_i^\infty.$$

The main difference between the 2-periodic and $2(p^n - 1)$ -periodic cases comes in translating these results into specific statements about ranks. In the 2-periodic case the answer is easy to state.

Corollary 7.4. *If E is the 2-periodic complete theory then $t(E)_G^*$ is concentrated in even degrees, and in each even degree it is free of rank $p^{nr} - p^{ir}$ over $E^0[1/u_i]$.*

For example when $n = 1$ and E is p -adic K-theory we obtain a sum of $(|G| - 1)$ copies of the p -adic rationals in each even degree and zero in each odd degree. From the existence of equivariant K-theory and its connection to representation theory, one may see this applies to any p -group [5, 3].

Corollary 7.5. *If E is the 2-periodic complete theory then $E_*(BG_+)$ is a sum of p^{ir} copies of E^0 in each even degree and $p^{nr} - p^{ir}$ copies of E^0/u_i^∞ in each odd degree.*

For example when $n = 1$ and E is p -adic K-theory we obtain \mathbb{Z}_p^\wedge in each even degree and a sum of $(|G| - 1)$ copies of \mathbb{Z}/p^∞ in each odd degree. Again, equivariant K-theory lets one see this applies to any p -group [2].

By way of contrast consider the $2(p^n - 1)$ -periodic case.

If $n = 1$ the coefficients of the Tate theory has a sum of $(|G| - 1)/(p - 1)$ copies of the p -adic rationals in each even degree and zero in odd degrees. Correspondingly $E_*(BG_+)$ is $\mathbb{Z}_{(p)}$ in each even degree, and a sum of $(|G| - 1)/(p - 1)$ copies of \mathbb{Z}/p^∞ in each odd degree.

8. APPENDIX: AN ALGEBRAIC PROOF IN THE 0 DIMENSIONAL CASE.

We give a more algebraic proof of Theorem 1.1 of [7] using the algebra of [3]. Since $E^0(BG_+)$ is a 0-dimensional Noetherian ring if E is Morava K-theory, it is enough to prove the following.

Theorem 8.1. *If G is a finite group and E is a v_n -periodic, complex oriented theory with $E^0(BG_+)$ Noetherian of Krull dimension 0, then $t(E)_G^* = 0$.*

It is enough to deal with the case when G is a p -group by using the transfer argument 6.3.

Suppose then that G is a p -group. From the local Tate spectral sequence (2-5) it is enough to show that $\check{H}_j^*(R^*) = 0$. We shall prove a more general algebraic statement; in fact the ring $R^* = E^*(BG_+)$ arising in our topological applications has certain completeness properties [3, 3.5] which show that $\check{H}_j^*(R^*) = \hat{H}_j^*(R^*)$, where $\hat{H}_j^*(\cdot)$ denotes local Tate cohomology. To obtain an algebraic result valid without completeness assumptions on R it is necessary to use local Tate cohomology.

Proposition 8.2. *If R is a 0 dimensional Noetherian ring then the local Tate cohomology at any ideal vanishes: $\hat{H}_j^*(R; M) = 0$*

Proof. Since $\hat{H}_j^*(R; M)$ is a module over $\hat{H}_j^*(R)$, it is sufficient to deal with the case $M = R$. We use the spectral sequence of the second avatar [3, 4.2]:

$$E_2^{*,*} = \check{H}_*^J(H_j^*(\Sigma R)) \implies \hat{H}_j^*(R).$$

Now if R is of dimension 0 the local cohomology is concentrated in degree 0, so that $H_j^*(R) = \Gamma_j(R)$, which is also finitely generated. Hence $J^e H_j^*(R) = 0$ for some e . The following lemma completes the proof of the proposition.

Lemma 8.3. *If R is a 0-dimensional Noetherian ring and N is a finitely generated module with $J^e N = 0$ for some e then $\check{H}_*^J(N) = 0$.*

Proof. In the 0-dimensional case the only possible nonzero Čech homology groups are in degrees 0 and -1 , which are the kernel and cokernel of the natural map $M \longrightarrow H_0^J(M)$ for any M . If M is Noetherian $H_0^J(M) = M_J^\wedge$, and the hypotheses on N guarantee that $N = N_J^\wedge$. \square

We note that the Rationality Theorem of [3] could similarly be used to give a substitute for Theorem 5.1. Indeed, provided v_i is a regular element on $E^*(BG_+)$ and $E^0(BG_+)$ is 1-dimensional, the Rationality Theorem shows that v_i is invertible on $\hat{H}_j^*(R)$. Since this is the E_2 term of a spectral sequence for calculating topological Tate cohomology, v_i is also invertible on $t(E)_G^*$.

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