Topology **Final**

1. Let $p : E \to B$ be a covering map and let $b_0, b_1 \in B$. Show that the sets $p^{-1}(b_0)$ and $p^{-1}(b_1)$ are in bijective correspondence.

2. Consider the circle as a subset of the complex plane, so $S^1 = \{z \in \mathbb{C} \mid |z| = 1\}$. Let $n \in \mathbb{Z}$ and define $g : S^1 \to S^1$ by $g(z) = z^n$. Recall that $\pi_1(S^1, 1) \cong \mathbb{Z}$, so $g_* : \mathbb{Z} \to \mathbb{Z}$ is a homomorphism of groups. Describe it. That is, if $k \in \mathbb{Z}$, what is $g_*(k)$?

3. Define $B^2 = \left\{\vec{x} \in \mathbb{R}^2 \mid \|\vec{x}\| \leq 1\right\}$. Let $f : B^2 \to B^2$ be continuous. Show that f has a fixed point—that is, a point $\vec{x} \in B^2$ such that $f(\vec{x}) = \vec{x}$.

4. What surface is represented by a 10-gon with edges identified in pairs as indicated by the recipe $abcdec^{-1}da^{-1}b^{-1}e^{-1}$?